

## **MBF 3C Unit 6 – Statistics – Probability – Outline**

Day	Lesson Title	Specific Expectations
1	Introduction to Probability	D2.1 D2.3
2	Theoretical Probability	D2.2
3	Theoretical Probability – Day 2	D2.2
4	Compare Experimental & Theoretical Probability	D2.4
5	Investigation using Technology – Comparing Experimental & Theoretical Probability	D2.5
6	Interpreting Statistics from the Media	D2.6
7	Review Day	
8	Test Day	
<b>TOTAL DAYS:</b>		<b>8</b>

- A2.1 – identify examples of the use of probability in the media and various ways in which probability is represented (e.g., as a fraction, as a percent, as a decimal in the range 0 to 1);
- A2.2 – determine the theoretical probability of an event (i.e., the ratio of the number of favourable outcomes to the total number of possible outcomes, where all outcomes are equally likely), and represent the probability in a variety of ways (e.g., as a fraction, as a percent, as a decimal in the range 0 to 1);
- A2.3 – perform a probability experiment (e.g., tossing a coin several times), represent the results using a frequency distribution, and use the distribution to determine the experimental probability of an event;
- A2.4 – compare, through investigation, the theoretical probability of an event with the experimental probability, and explain why they might differ (Sample problem: If you toss 10 coins repeatedly, explain why 5 heads are unlikely to result from every toss.);
- A2.5 – determine, through investigation using class-generated data and technology-based simulation models (e.g., using a random-number generator on a spreadsheet or on a graphing calculator), the tendency of experimental probability to approach theoretical probability as the number of trials in an experiment increases (e.g., “If I simulate tossing a coin 1000 times using technology, the experimental probability that I calculate for tossing tails is likely to be closer to the theoretical probability than if I only simulate tossing the coin 10 times”)  
(Sample problem: Calculate the theoretical probability of rolling a 2 on a number cube. Simulate rolling a number cube, and use the simulation to calculate the experimental probability of rolling a 2 after 10, 20, 30, ..., 200 trials. Graph the experimental probability versus the number of trials, and describe any trend.);
- A2.6 – interpret information involving the use of probability and statistics in the media, and make connections between probability and statistics (e.g., statistics can be used to generate probabilities).

<b>Unit 6 Day 1: Experimental Probability</b>		<b>MBF 3C</b>
	<b>Description</b> This lesson introduces the concept of probability, the different ways it can be represented (fraction, decimal or percent) and examines a probability experiment.	<b>Materials</b> -Three coins -Dice (for homework) -BLM 6.1.1
<b>Assessment Opportunities</b>		
<b>Minds On...</b>	<b>Whole Class → Discussion</b> Have the class indicate examples of how statistics are used in the media. Try to have them give a specific example of the value shown.  Here are some examples to get them started thinking... Based on the weather report for the current day, write the chances of rain/snow (depending on season) for today. If the newspaper/radio show from the morning indicated a sports team's chances of winning in an upcoming event, this could be written on the board. Anything of this nature could be used to start a discussion on where/how the media uses probabilities to represent information and the likelihood of something occurring.  Possible examples: In advertisements: 4 out of 5 dentists surveyed prefer our toothpaste. → Fraction 4/5 In news/weather: There is a 30% chance of rain/snow today. → Percent 30% In sports: "Sports personality" has a "batting average of" 0.343 → Decimal 0.343 (still indicates this person's chances of hitting the baseball)	
<b>Action!</b>	<b>Small Group → Investigation</b> Set up the following investigation: using 3 coins, toss all three coins. If all three are tails then you get 3 points and you try again – if any two are tails then you get 1 point and try again – if neither of the first two occur (i.e. only 1 or no tails) then you lose your turn and the next person tries. The first person to 15 points wins.  Have the students think about the chances of each of these situations happening. Both before and after the exercise, discuss with the students whether they think each situation is equally likely and what their opinions are about the way the points are awarded.  Following along with <b>BLM 6.1.1</b> , have them try the game in pairs and record the number of tails on each toss and the number of points awarded on each coin toss: 3, 1 or 0.  The activity is an example of a <b>probability experiment</b> . An experiment consists of a number of <b>trials</b> , essentially the number of times you had to toss the coins is the total number of trials for your experiment. For the above experiment there were specifically three possible <b>events</b> : <ul style="list-style-type: none"> <li>• getting 3 points</li> <li>• getting 1 point</li> <li>• getting no points</li> </ul> An <b>outcome</b> is defined as a specific and possible result from a trial of the experiment.  Once the coin tossing is complete, have the students set up a frequency distribution table and graph with the three events on the x-axis (0, 1, or 3 points) and the frequency of the outcome on the y-axis. Draw bars representing the frequency of each event.  Discuss the <b>experimental probability</b> (the probability of the event that arises from the experiment). The experimental probability is found by the ratio of the number of times a specific event occurs and the total number of trials.	

<p><b>Consolidate Debrief</b></p>	<p><b>Small Group → Think/Pair/Share</b></p> <p>Compare the results of the experimental probability between different groups. Were the results similar? Were they different? Why are they different? Could the experiment be changed or altered so that different results could be more similar.</p> <p>Try to lead the discussion into the ideas of tomorrow's topic – theoretical probability.</p>		
<p><i>Exploration</i></p>	<p><b>Home Activity or Further Classroom Consolidation</b></p> <p>Students complete BLM6.1.1.</p>		

## Experimental Probability

1. Perform an experiment to investigate the experimental probability of rolling a single die.
  - (a) Roll the die 10 times. Record the results of each roll
  - (b) Create a frequency distribution table and graph of the results of the 10 rolls.
  - (c) Determine the experimental probability of rolling a 1 after 10 rolls. Write this probability as a fraction, a decimal, and a percent.
  - (d) Roll the die another 40 times to make 50 rolls in total. Record the results of each roll.
  - (e) Create a frequency distribution table and graph of the results of all 50 rolls.
  - (f) Determine the experimental probability of rolling a 1 after 50 rolls. Write this probability as a fraction, a decimal, and a percent.
  - (g) Compare and contrast the two results of the two experimental probabilities.
  - (h) Are the results what you would expect?

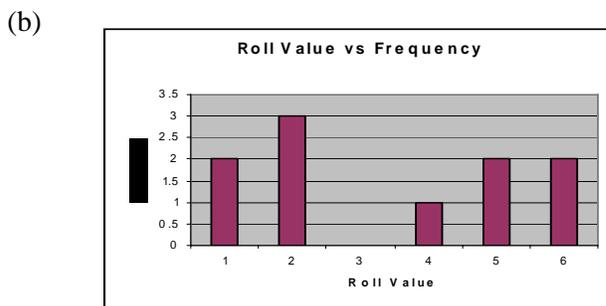
### Solutions:

Results will vary; have students compare their results.

Here is an example of possible results:

(a)

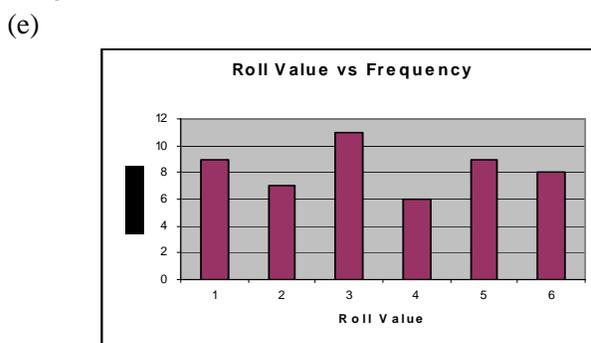
10 rolls Roll value	Frequency
1	2
2	3
3	0
4	1
5	2
6	2



(c) In the above example: Fraction:  $\frac{1}{5}$  Decimal: 0.20 Percent: 20%

(d)

50 Rolls Roll value	Frequency
1	9
2	7
3	11
4	6
5	9
6	8



(f) In the above example: Fraction:  $\frac{9}{50}$  Decimal: 0.18 Percent: 18%

(g) Answers here will vary → In this case with more rolls the probability dropped a little.

(h) Answers here will vary → Just honestly give your opinion about what you expected.

## Probability Experiment: Tossing Three Coins

Toss three coins. Record the number of tails of each toss in the table. Record the number of points: 3 points for all three tails, 1 point for any 2 tails, 0 points for 1 or no tails. If you receive 3 or 1 points in a turn, you get to go again. If you receive no points, switch to your partners turn. Play until one person reaches 15 points. (If you run out of space on the table, continue your points on the back)

**Your Points**

Toss #	# of Tails	Points
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		

**Partner's Points**

Toss #	# of Tails	Points
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		

Your total points: \_\_\_\_\_

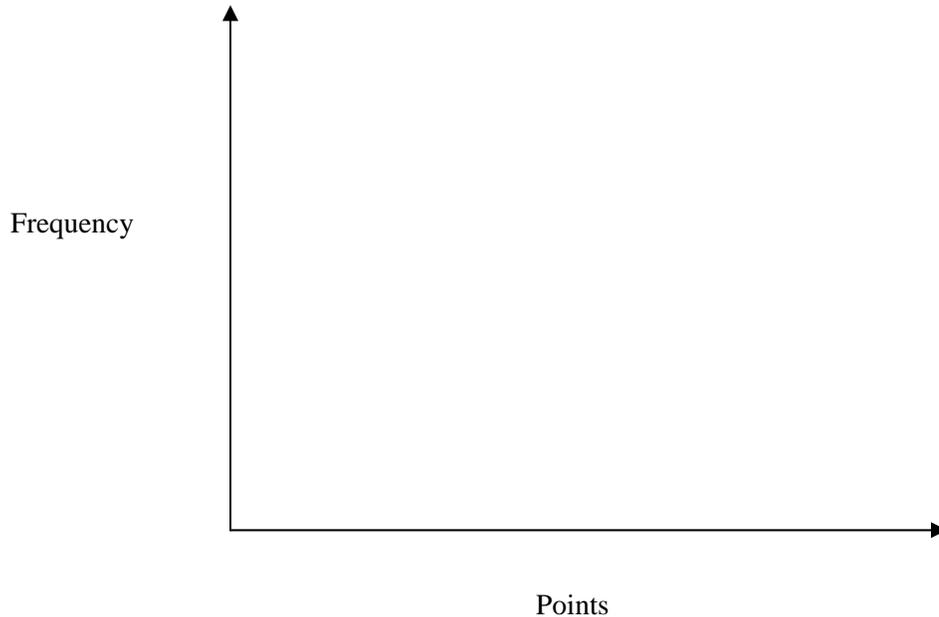
Partner's total points: \_\_\_\_\_

Create a frequency distribution using all of the rolls from you and your partner.

Event	Frequency (Number of times event occurs)
3 points	
1 point	
0 points	

## Probability Experiment: Tossing Three Coins

Create a frequency distribution graph.



Now using the frequency distribution table or graph, determine the **experimental probability** of obtaining 3 points, 1 point, or 0 points. The experimental probability is the ratio of the number of times an event occurs and the total number of trials.

Probability of Event A (3 Points):  $P(A) =$

Probability of Event B (1 Point):  $P(B) =$

Probability of Event C (0 Points):  $P(C) =$

<b>Unit 6 Day 2: Theoretical Probability</b>		<b>MBF 3C</b>
	<p><b>Description</b></p> <p>This lesson investigates theoretical probability and how to represent it in a variety of ways (fraction, decimal, percent).</p>	<p><b>Materials</b></p> <p>-Three coins -Dice and cards (for homework) BLM 6.2.1</p>
<b>Assessment Opportunities</b>		
<b>Minds On...</b>	<p><b>Whole Class → Discussion</b></p> <p>Review the results of the “rolling a die” experiment from the homework in the previous lesson, or perform the experiment together in the classroom.</p> <p>Discuss the following with the class: Were the results what you would have expected? What would you expect the results of the probability of rolling a 1 on a die to be? Why?</p> <p>The students should be able to determine that the expected result of rolling a 1 on a die is <math>\frac{1}{6}</math>, since each number on a die is equally likely.</p> <p>Discuss the expected probability of other events. What is the probability of getting a tail on a single coin? What is the probability of getting an Ace in a deck of cards?</p>	
<b>Action!</b>	<p><b>Whole Class → Teacher Directed</b></p> <p>To introduce the idea of <b>theoretical probability</b>, recall the results of yesterday’s experiment of flipping three coins.</p> <p>There are eight possible outcomes of flipping three coins. List out the possible outcomes, using T to show tails and H for heads.</p> <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;"> <ol style="list-style-type: none"> <li>1) T T T</li> <li>2) T T H</li> <li>3) T H T</li> <li>4) T H H</li> <li>5) H T T</li> <li>6) H T H</li> <li>7) H H T</li> <li>8) H H H</li> </ol> </div> <div> </div> </div> <p style="text-align: center; margin-top: 10px;">coin 1    coin 2    coin 3</p> <p>(You could spend time discussing organized counting methods or tree diagrams, if desired.)</p> <p>Now these eight possible outcomes can be grouped into the specific events we were looking for:</p> <p>Outcome #1 → was event <b>A</b> and earned 3 points,  Outcomes #2, 3, and 4 → were event <b>B</b> and earned 1 point, and  Outcomes #5, 6, 7, and 8 → were event <b>C</b> and earned no points, and lost our turn.</p> <p>Listing the outcomes and the events as shown above gives a more clear indication of your chances of having a particular event occur. In this case you can calculate the <b>theoretical probability</b> for each of the events, e.g. the theoretical probability of the first event above <b>A</b> is given as the total number of outcomes that match the event <i>over</i> the total number of <u>possible</u> outcomes. So in this case:</p>	

	<p> <math>P(A) = \frac{1}{8}</math> ← one way for the specific event to occur  <math>\frac{1}{8}</math> ← eight possible results for any trial </p> <p>and in general, the probability of any event <b>A</b> is given by the formula:</p> <p> <math>P(A) = \frac{n(A)}{n(S)}</math> where; <math>n(A)</math> represents the number of ways that event <b>A</b> can occur  and <math>n(S)</math> represents the number of total outcomes possible for the experiment. </p> <p>Now have the students try the same for events <b>B</b> and <b>C</b>.</p> <p>Answers:</p> <p> <math>P(B) = \frac{3}{8}</math> ← three ways for the specific event to occur  <math>\frac{3}{8}</math> ← eight possible results for any trial </p> <p> <math>P(C) = \frac{4}{8}</math> ← one way for the specific event to occur  <math>\frac{4}{8}</math> ← eight possible results for any trial  <math>= \frac{1}{2}</math> ← probabilities are commonly written in lowest terms </p> <p>Have the students re-write each of the probabilities above as a fraction:  <b><math>P(A) = 0.125</math></b>  <b><math>P(B) = 0.375</math></b>  <b><math>P(C) = 0.5</math></b> </p> <p>Now have the students re-write each of the probabilities above as a percent:  <b><math>P(A) = 12.5\%</math></b>  <b><math>P(B) = 37.5\%</math></b>  <b><math>P(C) = 50\%</math></b> </p> <p>Any of these three forms of representing the probability of an event is acceptable.</p>	
<b>Consolidate Debrief</b>	<p><b><u>Whole Class → Discussion</u></b></p> <p>When flipping 2 coins, what is the probability that you will get:</p> <p>(a) Only one head?  (b) Only one tail?  (c) Two heads?  (d) At least one tail?</p> <p>Answer:  There are four possible outcomes {HH, HT, TH, TT}</p> <p>(a) <math>2/4 = 1/2</math>  (b) <math>2/4 = 1/2</math> (Discuss similarities to (a))  (c) <math>1/4</math>  (d) <math>3/4</math> (Discuss similarities and differences to (c) – at least one tail means not both heads)</p> <div style="text-align: right;"> <pre> graph LR     A(( )) --- B(H)     A --- C(T)     B --- D(H)     B --- E(T)     C --- F(H)     C --- G(T)     D --- H1(HH)     E --- H2(HT)     F --- H3(TH)     G --- H4(TT) </pre> </div>	
<i>Application</i>	<p><b>Home Activity or Further Classroom Consolidation</b></p> <p>Students complete BLM 6.2.1</p>	

1. Find the probability of each of the following situations:
  - (a) You toss a coin → what is the probability of seeing tails come up?
  - (b) You toss two coins → what is the probability of seeing both coins show tails?
  - (c) You toss three coins → what is the probability of seeing only one tail on all three coins?
  - (d) You toss three coins → what is the probability of seeing at least one tail on all three coins?
  
2. Find the probability of each situation of rolling a six-sided die:
  - (a) What is the probability of rolling a 5?
  - (b) What is the probability of rolling a 1 or a 2?
  - (c) What is the probability of rolling an odd number?
  - (d) What is the probability of rolling a number greater than 2?
  
3. A standard deck of cards contains 52 cards – these cards are identified as follows: There are 4 suits: Spades, Hearts, Clubs and Diamonds. Each suit contains 13 cards: Ace (often valued at 1), numbered cards 2, 3, 4, 5, 6, 7, 8, 9, and 10, and then a Jack (“J”), a Queen (“Q”) and finally a King (“K”). Spades and Clubs are both black coloured cards and the Hearts and Diamonds are red coloured cards. The Jack, Queen and King cards are also often referred to as face cards as they have a face on them. Based on the above description of a standard deck of cards calculate the probability for the following situations – based on an experiment of drawing one card from a well-shuffle deck:
  - (a) What is the probability of drawing a red card from the deck?
  - (b) What is the probability of drawing a heart card from the deck?
  - (c) What is the probability of drawing an even numbered card (2, 4, 6, 8, 10 – of any suit) from the deck?
  - (d) What is the probability of drawing a face card from the deck?

**Solutions:**

**(Note:** after the first question only one method of showing the probability will be used -- any are acceptable.)

1. (a)  $1/2$ , **0.5**, or **50%**, (b)  $1/4$ , **0.25** or **25%**, (c)  $3/8$ , **0.375**, or **37.5%**, (d)  $7/8$ , **0.875**, or **87.5%**
2. (a)  $1/6$  (b)  $1/3$  (c)  $1/2$  (d)  $2/3$
3. (a)  $1/2$  (b)  $1/4$  (c)  $5/13$  (d)  $3/13$

<b>Unit 6 Day 3: Theoretical Probability Part 2</b>		<b>MBF 3C</b>																																																									
	<b>Description</b> This lesson provides further investigation of theoretical probability.	<b>Materials</b> -Deck of cards -2 dice BLM 6.3.1																																																									
<b>Assessment Opportunities</b>																																																											
<b>Minds On...</b>	<b>Whole Class → Demonstration</b> Using a deck of cards, review theoretical probabilities from the previous lesson.  From a full deck, what is the probability that you will remove the Queen of hearts? (1/52) What is the probability that you will remove any Queen? (4/52=1/13) What is the probability that you will remove a heart? (13/52 = 1/4) What is the probability of removing a red card? (26/52 = 1/2)  Now remove a card from the deck. You are left with 51 cards. Look at the card that is removed. Is it black or red? What value is it? How will this affect our results now that we know this specific card is removed?  Ask them what the probability is now of picking the Queen of hearts (assuming that card is still in the deck). (1/51) Repeat above questions with the new situation.																																																										
<b>Action!</b>	<b>Whole Class → Teacher Directed</b> We are going to look at some more involved questions regarding theoretical probability. The same concepts from yesterday still apply.  Roll two dice. What are the numbers that arise? What is the sum? How can we figure out the probability of rolling a specific sum?  Create an organized chart of possible outcomes for rolling a pair of dice. Provide students with the basics of the chart and have them fill in the sums.  <table border="1" style="margin-left: 20px;"> <thead> <tr> <th style="text-align: left;">Based on Sum of #'s</th> <th colspan="6" style="text-align: center;"># on 1<sup>st</sup> Die</th> </tr> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> </tr> <tr> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <th style="text-align: left;"># on 2<sup>ND</sup> Die</th> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> </tr> <tr> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> </tr> <tr> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> <td>11</td> </tr> <tr> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> <td>11</td> <td>12</td> </tr> </tbody> </table> Ask the student to reflect upon the game 7 or 11, and does it seem fair? Using the concept of theoretical probabilities from yesterday, have the students calculate the probabilities for the following events: <ul style="list-style-type: none"> <li>(a) Rolling an even sum</li> <li>(b) Rolling an odd sum</li> <li>(c) Rolling a sum of 7</li> <li>(d) Rolling a sum of 7 or less</li> <li>(e) Rolling a sum of more than 7</li> </ul>	Based on Sum of #'s	# on 1 <sup>st</sup> Die							1	2	3	4	5	6	1	2	3	4	5	6	7	2	3	4	5	6	7	8	# on 2 <sup>ND</sup> Die	3	4	5	6	7	8	9	4	5	6	7	8	9	10	5	6	7	8	9	10	11	6	7	8	9	10	11	12	
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	<p><i>Answers:</i></p> <p>(a) <math>P(A) = 18/36 \rightarrow 1/2</math></p> <p>(b) <math>P(B) = 18/36 \rightarrow 1/2</math></p> <p>(c) <math>P(C) = 6/36 \rightarrow 1/6</math></p> <p>(d) <math>P(D) = 21/36 \rightarrow 7/12</math></p> <p>(e) <math>P(E) = 15/36 \rightarrow 5/12</math></p>	
<p><b>Consolidate Debrief</b></p>	<p><b><u>Small Group → Creation</u></b></p> <p>Have the students create their own probability questions, either using situations that have been covered as a class (cards, dice, coins) or using a new situation of their choice. Have students present their question to the class and have the class answer it.</p> <p>Discuss the different questions as you go to further refine the concept of probability.</p>	
<p><i>Application Concept Practice</i></p>	<p><b>Home Activity or Further Classroom Consolidation</b></p> <p>Students complete BLM 6.3.1</p>	

1. Using a standard deck of cards, consider the following possibilities:
  - (a) What is the probability of picking a 7, 8, or 9?
  - (b) What is the probability of picking a heart or a face card?
  - (c) If the deck is now split for the game of Euchre (only 9, 10, J, Q, K, A exist in the deck; all other cards are removed) what is the probability of picking an Ace?
  
2. Using the table of possible sums from rolling a pair of dice answer the following questions:
  - (a) What is the probability of rolling sum that is a multiple of 3?
  - (b) What is the probability of rolling sum that is a multiple of 5?
  - (c) What is the probability of rolling sum that is 7 or 11?
  - (d) Ignoring the sums for this question what would be the probability of rolling doubles? (Doubles occur when both numbers on the die match – i.e. 1<sup>st</sup> die shows a 1 and so does the 2<sup>nd</sup>.)
  
3. Jesse needs to get ready for school. He has two pair of pants to choose from: one black and one brown. He has three shirts to choose from: one red, one green, and one white. Any combination of pants and shirts is equally likely. (hint: use a tree diagram)
  - (a) What is the probability that he will wear the green shirt and the brown pants?
  - (b) What is the probability he will wear the black pants with any of the shirts?

**6.3 Homework Solutions:**

1. (a)  $\frac{3}{13}$  (b)  $\frac{11}{26}$  (c)  $\frac{1}{4}$
2. (a)  $\frac{1}{3}$  (b)  $\frac{7}{36}$  (c)  $\frac{2}{9}$  (d)  $\frac{1}{6}$
3. (a)  $\frac{1}{6}$  (b)  $\frac{1}{2}$

<b>Unit 6 Day 4: Experimental and Theoretical Probability</b>		<b>MBF 3C</b>															
	<p><b>Description</b></p> <p>Experimental and Theoretical Probability          Compare the theoretical probability of an event with the experimental probability.          Explain why they might differ.</p>	<p><b>Materials</b></p> <p>Decks of cards,          graphing          calculators,          copies of BLM          6.4.1 and 6.4.2</p>															
<b>Assessment Opportunities</b>																	
<b>Minds On...</b>	<p><b>Whole Class → Demonstration</b></p> <p>The teacher can have the letters SESAME on card stock and placed in a paper bag. State the probability of each event if one letter is selected at random from the word SESAME.          a) an “s”    b) an “m”    c) a vowel    d) a consonant    e) an “s” or an “e”</p> <p><b>Solution:</b></p> <table style="width: 100%; border: none;"> <tr> <td style="text-align: center;">a)</td> <td style="text-align: center;">b)</td> <td style="text-align: center;">c)</td> <td style="text-align: center;">d)</td> <td style="text-align: center;">e)</td> </tr> <tr> <td style="text-align: center;"><math>\frac{2}{6}</math></td> <td style="text-align: center;"><math>\frac{1}{6}</math></td> <td style="text-align: center;"><math>\frac{3}{6}</math></td> <td style="text-align: center;"><math>\frac{3}{6}</math></td> <td style="text-align: center;"><math>\frac{4}{6}</math></td> </tr> <tr> <td style="text-align: center;"><math>\frac{1}{3}</math></td> <td style="text-align: center;"><math>\frac{1}{6}</math></td> <td style="text-align: center;"><math>\frac{1}{2}</math></td> <td style="text-align: center;"><math>\frac{1}{2}</math></td> <td style="text-align: center;"><math>\frac{2}{3}</math></td> </tr> </table> <p>Have the students discuss in pairs if the answer to letter e) seems reasonable to them.</p>	a)	b)	c)	d)	e)	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{3}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	
a)	b)	c)	d)	e)													
$\frac{2}{6}$	$\frac{1}{6}$	$\frac{3}{6}$	$\frac{3}{6}$	$\frac{4}{6}$													
$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$													
<b>Action!</b>	<p><b>Whole Class → Teacher Directed</b></p> <p><b>Experimental and Theoretical Probability Mini-Lesson:</b></p> <p>Often in the world of business, the theoretical probability of an event is used to conduct simulations (experiments) which estimate future outcomes.</p> <p><b>Theoretical Probability:</b>      This is the mathematical probability that can be calculated without actually doing the activity.</p> <p><b>Experimental Probability:</b>      The probability that you have observed when performing an experiment.</p> <p style="text-align: center;">=      <math>\frac{\text{number of times that the event occurred}}{\text{number of trials}}</math></p> <p><b>Example 1:</b> One percent of the tires produced on a manufacturer’s assembly line are defective.</p> <p>a) What is the probability that the next 5 are defective?</p> <p style="padding-left: 40px;">The probability that one is defective is <math>\frac{1}{100}</math>.</p> <p style="padding-left: 40px;"><math>\therefore</math> the P(5 defective) = <math>\left(\frac{1}{100}\right)^5 = \frac{1}{10000000000}</math></p> <p>b) Describe an experiment to determine the experimental probability of this event (i.e. how could an average person estimate this probability?).</p> <p style="padding-left: 40px;">We need to use something concrete which can be split 100 ways. Often, the quickest method is to create a spinner.</p>																

**Steps for making a spinner with a theoretical prob. of 1/100.**

1. Create a spinner where 1 piece out of 100 is coloured red and the rest are coloured black. The red piece represents the 1 defective piece.
2. To correctly cover  $\frac{1}{100}$  of the circle, the centre angle of the red piece should be  $3.6^\circ$  since  $\frac{1}{100} \times 360^\circ = 3.6^\circ$ .
3. Spin it. Record whether or not it lands on red.
4. Repeat 4 more times to simulate the 5 tires.
5. Repeat step 4 100 times because a good simulation should always have many trials.

**Example 2:** Suppose that you toss 10 coins at once, repeatedly.

- a) How many heads should occur in each toss?

$$P(\text{head}) = \frac{1}{2}$$

$\therefore$  the number of heads = P(head)  $\times$  Number of trials

$$= \frac{1}{2} \times 10$$

$$= 5$$

- b) Explain why you will not necessarily see 5 heads every time.

Probabilities are like averages. If the experiment is done enough times, there

will be a  $P(\text{head}) = \frac{1}{2}$ .

See BLM 6.4.1 handout entitled: **Investigation 6.4**

On the page Item #2.  $\rightarrow$  Reasoning for theoretical probability of passing:

Number Correct	Probability	Pass/Fail
0	5.6314%	Fail
1	18.7712%	Fail
2	28.1568%	Fail
3	25.0282%	Fail
4	14.5998%	Fail
5	5.8399%	Pass
6	1.6222%	Pass
7	0.3090%	Pass
8	0.0386%	Pass
9	0.0029%	Pass
10	0.0001%	Pass

Adding the Pass results gives a probability of passing of 7.8%

**Consolidate  
Debrief**

**Pairs – Brainstorm**

Have the students brainstorm the types of games that use probability (ie board games where you roll a die, casino games)

*Exploration*

**Home Activity or Further Classroom Consolidation**

Students complete BLM 6.4.2.

## Investigation

Suppose that your final exam has 10 multiple choice questions, each with possible answers of A, B, C or D.

1. Use a deck of cards to simulate the probability of passing this portion of the exam simply by guessing.
  - a) Choose a suit to represent the correct answer. Name it here: \_\_\_\_\_
  - b) Draw 10 cards one after the other, replacing each card between drawings.
  - c) Record whether or not you passed in the chart below.
  - d) Repeat all 10 draws 19 more times.

Trial	Number of Cards of Chosen Suit (out of 10)	Pass or Fail
	<i>Experimental Probability</i> = $\frac{\quad}{10}$	
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

2. Calculate your experimental probability of passing the test.
3. Given that the theoretical probability that you would pass is 7.8%, how close were you to this value?
4. Explain why your simulation may not have been exact.

### Experimental and Theoretical Probability

1. The centre on a high school basketball team has a free-throw success rate of 85%. Describe a simulation that would help to determine the probability that he might miss 5 shots in a row. Be specific in your description.
2. Describe a simulation that would estimate the probability of having 3 girls in a family of 3 children. Be specific in your description.
3. An event occurs every 8 out of 19 times on average during a simulation. What is the experimental probability of this event?
4. A coin is flipped 15 times to simulate a family having a boy. Heads were used to represent boys and tails for girls. Twelve heads were recorded. Is the experimental probability of  $\frac{4}{5}$  close to the theoretical probability?
5. A casino introduces a new game called *Test Your Luck*. To play, you draw a card from a deck. If it's a heart, they will match your bet. If it's black, they keep your bet. Explain why this is not a good game to play.

### Solutions

1. Make a spinner with sector angles 306E and 54E. Spin it 5 times recorded the number of times that it hits the 54E section. Repeat 100 times. 2. Flip a coin 3 times. Assign heads to girls.

Record whether all 3 are girls. Repeat 100 times. 3.  $\frac{8}{19}$  4. No, the theoretical prob. is  $\frac{1}{2}$ . 5.

The probability of losing is twice the probability of winning.

<b>Unit 6 Day 5: Probability</b>		<b>MBF 3C</b>
	<p><b>Description</b></p> <p>Investigating Probabilities Determine, through investigation, the tendency for the experimental probability to approach the theoretical probability after many trials.</p>	<p><b>Materials</b></p> <p>Coins, Graphing Calculators (or Excel), BLM6.5.1</p>
<b>Assessment Opportunities</b>		
<b>Minds On...</b>	<p><b><u>Pairs → Discussion</u></b></p> <p>Ask the class to discuss in pairs if it is reasonable that a die when tossed 100 times should give twenty-five 3's as an answer.</p> <p>How many times should a die show a 2 if it is tossed a) 10 times?                      b) 100 times?                      c) 1000 times?</p> <p><b><u>Solution:</u></b></p> <p>a) Number of times = Probability × number of trials      <math>= \frac{1}{6} \times 10</math>  <math>\approx 1.7</math></p> <p>b) <math>= \frac{1}{6} \times 100</math>                      c) <math>= \frac{1}{6} \times 1000</math>  <math>\approx 16.7</math>                              <math>\approx 166.7</math></p> <p>In pairs ask the class this scenario seems reasonable “Peggy reported that she tossed a die 1000 times and came up with the number 4 three hundred times” What could possibly explain this occurrence?</p>	
<b>Action!</b>	<p><b><u>Whole Class → Teacher Led Lesson</u></b></p> <p><b><u>Investigating Probabilities Investigation:</u></b></p> <p><b>Part One: Together</b></p> <p>Each week, you and your brother fight over who will pay the \$25 for gas. You decide to toss 2 coins. If they are either both H or both T, you win and your brother pays. If they are different, you pay.</p> <p>A) Have students pair up and flip two coins after assigning one student as the player who wins with HH or TT.</p> <p>B) Have them record who pays.</p> <p>C) Repeat 3 more times to simulate a 4-week month.</p> <p>D) Collect info from 10 pairs and complete the chart together.</p>	

		<table border="1"> <thead> <tr> <th>Month (Team)</th> <th>HH/TT wins</th> <th>HH/TT losses</th> <th>Cost for HH/TT player</th> </tr> </thead> <tbody> <tr><td>1</td><td></td><td></td><td></td></tr> <tr><td>2</td><td></td><td></td><td></td></tr> <tr><td>3</td><td></td><td></td><td></td></tr> <tr><td>4</td><td></td><td></td><td></td></tr> <tr><td>5</td><td></td><td></td><td></td></tr> <tr><td>6</td><td></td><td></td><td></td></tr> <tr><td>7</td><td></td><td></td><td></td></tr> <tr><td>8</td><td></td><td></td><td></td></tr> <tr><td>9</td><td></td><td></td><td></td></tr> <tr><td>10</td><td></td><td></td><td></td></tr> </tbody> </table>	Month (Team)	HH/TT wins	HH/TT losses	Cost for HH/TT player	1				2				3				4				5				6				7				8				9				10					
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	<p><b>Part Two: Investigation (alone or in pairs)</b>  See BLM 6.5.1 handout entitled: <i>Investigation 6.5</i></p>																																															
<b>Consolidate Debrief</b>	<p><b><u>Whole Class → Discussion</u></b></p> <p>Collect the Investigation results. Discuss the answers as a class.</p>																																															
	<p><b>Home Activity or Further Classroom Consolidation</b></p>																																															

MBF 3C  
BLM 6.5.1

Name(s): \_\_\_\_\_  
Date: \_\_\_\_\_

## Investigation

1. Is the coin toss for gas a fair game? Explain.
2. How much should you expect to pay in a 4-week month?
3. What is the probability that you will never pay in a 4-week month? Show your work.
4. What is the probability that you will pay at least once in a 4-week month? Show your work.

## Investigation

5. Repeat the in-class simulation on the graphing calculator. *Record your results on the next page.*

- Steps:
- A) Go to the **MATH** function. Cursor over to the **PRB** menu (<<<).
  - B) Select **5:randInt** to access the random integer function.
  - C) Assign 0 for heads and 1 for tails.
  - D) On your screen you will see **randInt(**  
Type **0,1,2)** It is important to include the commas as written.  
This tells the calculator to create lists of the numbers 0 and 1, 2 at a time.
  - E) Press **ENTER** four times to simulate a 4-week month.
  - F) Record your results in the chart provided. Remember, you win if you see either {0 0} or {1 1}.
  - G) Repeat 23 times.

Month	Wins	Losses	Cost
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			

6. What was your average cost using the results in Question 5?

## Investigation

7. How does your answer to Question 6 compare with your answer to Question 2?
  
8. Using the results in Question 5, how often did you never pay in a month?
  
9. How does your answer to Question 8 compare with Question 3?
  
10. Explain your answers to Question 7 and Question 9.

### **Simulation 2**

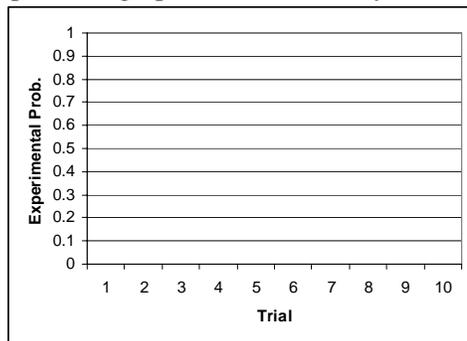
11. What is the probability of rolling a 2 on a numbered cube?
  
  
  
  
  
  
  
  
  
  
12. Use the randInt function on the graphing calculator with the numbers (1,6,10) to generate 10 numbers between 1 and 6.

## Investigation

13. Record how many of each set of 10 are 2's. Use the chart below. Repeat by pressing ENTER until you've completed the chart.

Trial	Number of 2's	Experimental Probability $\frac{\text{\# of 2's}}{10}$
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

14. Complete the graph and describe any trends.



## Investigation

### Alternate instructions for step 5 – for TI-89 Titanium Graphing Calculator

5. Repeat the in-class simulation on the graphing calculator. *Record your results on the next page.*

Steps: A) *You need to be in the “Home” window*

B) *Go to the **MATH** option list. (2nd button and the number 5)*

C) *Select the **Probabilty** submenu (item 7:).*

D) *Then select **rand**( (item 4:)*

*Type           4)           ← type a 4 and then a close-bracket*

*This function will generate a random integer in this case either 1, 2, 3 or 4.*

E) *Press **ENTER** four times to simulate a 4-week month.*

F) *Record your results in the chart provided Remember, you win if you see either a 1 or 4, and lose if you see a 2 or 3.*

G) *Repeat 23 times.*

### Alternate instructions for step 12 – for TI-89 Titanium Graphing Calculator

12. Use the graphing calculator to generate random numbers between 1 and 6 using **rand(6)** this will generate a random number between 1 and 6. You will have to hit **ENTER** 10 times to simulate the 10 rolls and count the number of 2's in the 10 rolls. Then repeat this 9 more times to complete the simulation.

<b>Unit 6 Day 6: Probability and Statistics in the Media</b>		<b>MBF 3C</b>
	<p><b>Description</b></p> <p>This lesson has students interpreting information using probability and statistics in the media, and making connections between probability and statistics.</p>	<p><b>Materials</b></p> <p>Computer Lab with access to internet for BLM 6.6.1 OR pre-notification of students to bring in newspaper and magazine articles containing statistics and/or probability items. BLM 6.6.2</p>
<b>Assessment Opportunities</b>		
<b>Minds On...</b>	<p><b>Whole Class → Discussion</b></p> <p>Begin a discussion about where students have seen statistics and/or probability in the media.</p> <p><i>Possible examples to aid in getting the discussion going:</i></p> <p><b>TV</b> → The reality TV shows like “American Idol” use the statistics to decide who to let go from the show. Many people make predictions as to who they think will be taken off the show but these are not often based on the statistics or probabilities.</p> <p><b>Weather</b> → Weather predictions are often made through statistical analysis of past weather and similarity to current conditions. Based on the analysis of these statistics a probability is generated for the type of weather that is to be expected.</p> <p>Try to indicate where the probability predictions are coming from – in many cases it should be coming from some form of statistical analysis.</p>	
<b>Action!</b>	<p><b>Whole Class → Investigation</b></p> <p>Follow BLM 6.6.1 with the students using either the newspaper/magazine articles that contain statistics/probability articles or computers with access to the internet.</p> <p>Observe and help students with finding and analyzing the articles/internet sites. If many are still having difficulty it might be helpful to break and move to the Consolidate Debrief section to outline with the students where the investigation BLM 6.6.1 is leading.</p> <p>GM’s OnStar service is a wireless communication system between a central call centre and an OnStar enabled car, allowing drivers to call for help or ask for other services. The volume of monthly OnStar activity reported by Richard Russell at <a href="http://en.autos.sympatico.msn.ca/guides_and_advice/article.aspx?cp-documentid=635432">http://en.autos.sympatico.msn.ca/guides_and_advice/article.aspx?cp-documentid=635432</a> is as follows:</p> <ul style="list-style-type: none"> <li>• 1,000 airbag deployment calls</li> <li>• 300 crash notifications</li> <li>• 11,000 emergency service calls</li> <li>• 5,400 'Good Samaritan' calls; Orange Alerts; someone in trouble etc.</li> <li>• 325 stolen vehicle location assists</li> <li>• 41,000 remote unlock calls</li> <li>• 24,000 requests for roadside assistance</li> <li>• 329,000 requests for route assistance</li> </ul>	

	<p>Total number of calls: 412,025</p> <p>Using the above list of statistics on GM's OnStar calls create a list of probabilities for each type of call.</p> <p>Using these probabilities answer the following questions:</p> <ol style="list-style-type: none"> <li>1) If you were an OnStar Operator what would be the probability of the next call being a person needing some roadside assistance?</li> <li>2) If you were an OnStar Operator what would be the probability of the next call being a person needing their car unlocked?</li> <li>3) What type of call has the highest probability? What is it?</li> <li>4) What type of call has the lowest probability? What is it?</li> <li>5) Based on what you've seen in the media what type of headline might you see in a newspaper based on the statistics given above?</li> <li>6) How accurate might those headlines be based on what you know about statistics?</li> </ol> <p>Answers:</p> <ol style="list-style-type: none"> <li>1) about 6% → (5.82% is more accurate)</li> <li>2) about 10% → (9.95% is more accurate)</li> <li>3) Route assistance or asking for directions. → about 80% → (79.85% is more accurate)</li> <li>4) Car crashes → 0.07% → very unlikely.</li> <li>5) Many possibilities: e.g. "GM Car owners are driving lost 80% of the time." "GM Car owners lock their keys in their car 10% of the time." "GM Car owners have fewer accidents."</li> <li>6) Not very → Since there is not enough information. You don't know the population of GM car owners vs. other types of cars.</li> </ol>	
<p><b>Consolidate Debrief</b></p>	<p><b><u>Whole Class → Discussion</u></b> Ask the students to reflect upon the saying "You shouldn't believe everything you hear" Ask them how this relates to probability.</p>	
<p><i>Application</i></p>	<p><b>Home Activity or Further Classroom Consolidation</b></p> <p>Students complete BLM 6.6.1 or BLM6.6.2.</p>	

1. You've spent some time learning about statistics and how they are calculated and used in the media. If you were a statistician and wanted to relate to people some information about a topic of interest to you what steps would you have to follow in order to be able to report your findings?
2. Find two more examples of statistics in the media and generate another set of at least three questions based on the information you analyse from the statistics.
3. Generate at least two possible headlines for newspaper articles based on your statistic examples.
4. Find two examples of statistics in the media being used to mislead or misrepresent statistical information.

**Solutions:**

1. Answers may vary depending on the topic chosen: e.g. Topic idea: "Wish to report on number of car crashes at a particular intersection in a particular town." First step do some research on the particular intersection → police reports (if possible) about accidents at that intersection, include variables in the study → weather conditions, number of cars in crash, time of day, direction of vehicles involved, direction of the vehicle charged in the incident. Next step: Perform survey of the intersection, count traffic through the intersection, number of vehicles, direction of vehicles, time of day, if any crashes occur during the survey include relevant data in this survey. Next step: analyse the results, Based on the results report the information learned to the appropriate people.
2. Answer will vary again based on articles found → results will be similar to the investigation performed in class as well as the example based on GM OnStar statistics outlined in class as an example.
3. Answer will vary based on articles → results will be similar to the investigation performed in class as well as the example based on GM OnStar statistics outlined in class.
4. Answer will vary based on articles → you should be able to describe how the media has misrepresented their information. Possible methods are: using inappropriate graph types for the information given, using sample sizes that are too small for the population they are attempting to represent, not giving enough information like total number of people involved in the survey compared to the population being represented, not using proper scales on the axes of the graph given, etc.

## **Investigation: Probability in the Media**

Based on the discussions begun in class, search through your example(s) of media sources for examples of statistical information. (*Either newspapers, magazines, or the internet.*)

Once you have at least three examples you feel you can use, do the following for each:

- 1) Analyse the statistical information in your examples. E.g. find the mean, median and mode if appropriate, using totals and specific values to generate the probabilities of those specific values occurring, etc.
- 2) From this analysis generate some questions that could be used to get another student to analyse your statistical example. You should come up with at least three questions for each of your statistical examples.
- 3) Find a partner and share your statistic examples and questions. You work on their questions while they work on yours. Once you've completed the questions, discuss possible results of your analysis – e.g. develop appropriate headlines for newspaper articles based on your findings.